Reading

Brassard & Bratley, Chapter 6.

Practice

Brassard & Bratley, 6.2, 6.4, 6.5, 6.9, 6.10, 6.11, 6.12, 6.18, 6.19, 6.20.

To Be Handed In

4. The \( n \) cities, \( c_1, c_2, \ldots, c_n \), along a straight highway are to be covered by a series of cell towers to be placed along the highway. A cell tower covers all cities that are within \( R \) miles of it. Let \( d_i \) be the distance (in miles) between \( c_i \) and \( c_{i+1} \) for \( i = 1, \ldots, n - 1 \). Give an efficient algorithm for placing the minimum number of towers that cover all of the cities. Show your algorithm is correct and analyze its running time. What happens if the highway isn’t straight? Does your algorithm still work? If so, prove it. If not, give a counterexample.

5. You own \( n \) stocks each with current value $100 that you must sell. You know that the \( i \)th stock is appreciating at a rate of \( r_i \) percent per day. For example, if \( r_1 = .1 \) then after one day the first stock will be worth $110, after two days, $121, etc. You plan on selling one stock a day until they are all sold. Give an algorithm for deciding the order to sell the stocks that maximizes your profit. Prove your algorithm is correct and analyze its running time. What if the \( i \)th stock’s value is shrinking at a rate \( r_i \), e.g., if \( r_1 = .5 \) then after the first day the first stock is worth $50, after two days, $25, etc., and you wish to minimize your losses selling one stock per day. Does a simple greedy strategy work in this case? If so, prove it. If not, give a counterexample.

Bonus

\( P \) prisoners are to be lined up against a wall in such a way that prisoner \( i \) can see prisoners \( i - 1 \) down to 1 but not the prisoners behind him. Each has either a black or white hat on but doesn’t know which. Starting with the last prisoner each has to declare the color of their hat. If correct they live, if not they die. Assume the order of the prisoners is chosen randomly. Find a scheme for the prisoners to maximize the number that live.