COMP 312
Assignment 8
Due at 9 a.m., Tuesday, December 4, 2007
All problems are of equal value.

Reading

Brassard & Bratley, Chapter 13, with emphasis on sections 13.1-3.

Practice

Brassard & Bratley, 13.2, 13.3(**), 13.4, 13.5, 13.6, 13.8, 13.11, 13.13.

To Be Handed In

1. Brassard & Bratley 13.1.
2. Brassard & Bratley 13.9.
3. Show there is a polynomial time algorithm which finds a 6-coloring of any planar graph. (Make sure you show your algorithm runs in polynomial time by giving an upper bound on its running time.) You may use the fact that any planar graph has a vertex of degree 5 or less.
4. Show how to color a planar graph using at most two times the optimal number of colors required to color graph, i.e., give a 2-approximation algorithm for the problem of coloring a planar graph.
5. Given a graph G an *independent set* is a set of nodes of G such that there is no edge between any pair of nodes in the set. Assume there is a weight associated with each node of G. The weight of an independent set is the sum of the weights of the nodes in the set.
   (a) Show the following problem is NP-complete: Given a graph G and an integer k decide if G has an independent set of weight greater or equal to k. You may assume that the problem of deciding if a graph G has an independent set of size k is NP-complete.
   (b) Describe a greedy algorithm for finding an independent set of large weight.
   (c) Show that your algorithm finds an independent set with weight at least $\frac{1}{d}$ times the optimal (maximum weight) independent set where d is the maximum degree of any node in G. (Hint: Let X be the independent set your algorithm finds and let Y be an optimal independent set. Show that for any $v \in Y$ either $v \in X$ or there is a $u \in X$ such that (u, v) is an edge and the weight of u is greater or equal to the weight of v.)

Bonus

You have 10 bags. How many marbles do you need so that you can have a different number of marbles inside each bag?