COM 312
Assignment 7
Due at 9 a.m., Tuesday, December 8, 2009
All problems are of equal value.

Reading

Cormen, Leiserson, Rivest, Stein, Chapters 35 (with emphasis on sections 35.1-3).

Practice

CLRS, 35.1-1..5, 35.2-1...4, 35.3-1..5, 35-1, 35-2, 35-3.

To Be Handed In

1. (a) Let G be a graph with (positive) edge weights that do not satisfy the triangle inequality and let C be the maximum weight edge in G. Show that if you construct G' from G by adding C to all of the edge weights of G, then G' satisfies the triangle inequality and any optimal TSP of G is an optimal TSP of G' and vice versa.

   (b) What is wrong with the following argument that $P = NP$? Given a general TSP problem we can use part (a) to convert it to a TSP where the triangle inequality holds and then we can approximate it to within a factor of 2 using Theorem 35.2. But this contradicts Theorem 35.3 unless $P = NP$.

2. CLRS, 35-4

3. A k-coloring of a graph G is an assignment of numbers (colors) from 1 to k to each vertex of G such that adjacent vertices are assigned different colors. Show there is a polynomial time algorithm which finds a 2-coloring of a graph if the graph can be colored with 2 colors.

4. Show there is a polynomial time algorithm which finds a 6-coloring of any planar graph. A graph is planar if it can be drawn on the plane in such a way that none of its edges cross. Make sure you show your algorithm runs in polynomial time by giving an upper bound on its running time. You may use the fact that any planar graph has a vertex of degree 5 or less.

5. Show how to color a planar graph using at most two times the optimal number of colors required to color graph, i.e., give a 2-approximation algorithm for the problem of coloring a planar graph. You may use the fact that any planar graph can be colored with at most 4 colors.