COMP 312
Assignment 6
Due at 9 a.m., Thursday, April 28, 2011

All problems are of equal value.

Reading

Cormen, Leiserson, Rivest, Stein, Chapter 35.

Practice

CLRS, 35.1-1..4, 35.2-1..3, 35.2-5, 35.3-1..5, 35.4-1..3, 35.5-1..4, 35-2..7.

To Be Handed In

1. CLRS, 35.1-5
2. CLRS, 35.2-4
3. CLRS, 35-1
4. (a) A k-coloring of a graph G is an assignment of numbers (colors) from 1 to k to each vertex of G such that adjacent vertices are assigned different colors. Show there is a polynomial time algorithm which finds a 2-coloring of a graph if the graph can be colored with 2 colors.

   (b) Show there is a polynomial time algorithm which finds a 6-coloring of any planar graph. A graph is planar if it can be drawn on the plane in such a way that none of its edges cross. Make sure you show your algorithm runs in polynomial time by giving an upper bound on its running time. You may use the fact that any planar graph has a vertex of degree 5 or less.

   (c) Show how to color a planar graph using at most two times the optimal number of colors required to color the graph, i.e., give a 2-approximation algorithm for the problem of coloring a planar graph. You may use the fact that any planar graph can be colored with at most 4 colors.
5. (a) Describe an $O(n^2)$ time algorithm to find the size of the largest clique in an n node graph when all of the vertices are of degree either $n-1$ or $n-2$.

   (b) Show by induction that the solution to the following recurrence is $O(1.47^n)$:

   $$T(n) = c_0,\ n \leq 4; \quad T(n) = c_1 \cdot n^2 + T(n-1) + T(n-3).$$

   (c) Describe an $O(1.47^n)$ time algorithm for finding a clique in an arbitrary n node graph. Prove the correctness and running time of your algorithm.