COMP 312
Assignment 8
Due at 9 a.m., Thursday, April 28, 2005
All problems are of equal value.

Reading
Brassard & Bratley, Chapter 13, sections 13.1-3.

Practice

To Be Handed In
5. Given a graph $G$ an independent set is a set of nodes of $G$ such that there is no edge between any pair of nodes in the set. Assume there is a weight associated with each node of $G$. The weight of an independent set is the sum of the weights of the nodes in the set.
   
   (a) Show the following problem is NP-complete: Given a graph $G$ and an integer $k$ decide if $G$ has an independent set of weight greater or equal to $k$. You may assume that the problem of deciding if a graph $G$ has an independent set of size $k$ is NP-complete.

   (b) Describe a greedy algorithm for finding an independent set of large weight.

   (c) Show that your algorithm finds an independent set with weight at least $\frac{1}{d}$ times the optimal (maximum weight) independent set where $d$ is the maximum degree of any node in $G$. (Hint: Let $X$ be the independent set your algorithm finds and let $Y$ be an optimal independent set. Show that for any $v \in Y$ either $v \in X$ or there is a $u \in X$ such that $(u, v)$ is an edge and the weight of $u$ is greater or equal to the weight of $v$.)

Bonus
Three gamblers enter a room and a red or blue hat is placed on each gambler’s head. The color of each hat is determined by a random coin toss. Each gambler can see the other’s hats but not their own. Once they have had a chance to look at the other hats, each gambler must simultaneously guess the color of their own hat or decide to pass (i.e., make no guess). If at least one gambler guesses correctly and no gambler guesses incorrectly, they share a $3,000,000 prize. No communication of any sort is allowed, except for an initial strategy session before the game begins. Give a strategy that maximizes their chance of winning.